We report parity-time (PT) symmetry breaking in electrically injected, coherently coupled, vertical cavity surface emitting laser arrays. We predict beam steering, mode evolution, and mode hopping as consequences of the non-Hermiticity of the array, analyzed by the temporal coupled-mode theory with both an asymmetric gain distribution and local frequency detuning. We present experimental confirmations of the predicted mode evolution, mode hopping, and PT symmetry breaking with quantitative agreement with the theory.

1. INTRODUCTION

The optical realization of non-Hermitian parity-time (PT) symmetric systems has drawn great interest in recent years [1–21]. An optical system with a symmetric index profile and an antisymmetric gain/loss profile formally exhibits PT symmetry by an analogy to the PT symmetry in quantum mechanics [1,2]. One realization of this index (and gain/loss) profile is two coupled waveguides or resonators, where one of the waveguides or resonators exhibits optical gain, and the other one is lossy [6,18–20]. PT symmetry breaking, exceptional points [6,16,18–20,22,23], unidirectional light propagation [12,19], and side-mode suppression [13,17,18], have been previously experimentally demonstrated. The optical gain in the majority of the systems above is provided by optical pumping. Below, we discuss electrically injected diodes, which are appealing and necessary for practical applications in integrated photonics. Photonic integrated circuits are currently under development because of their significant performance advantages in many existing and new deployment areas, such as data centers, smartphones, and autonomous automobiles. Compared to optically pumped PT-symmetric coupled resonators, an electrically pumped system presents new and interesting physical effects. For example, selective pumping perturbs the index profile through the carrier and thermal effect in addition to the gain/loss profile, thus generating frequency detuning. We propose the temporal coupled-mode theory as a means to analyze and make quantitative predictions of optically coupled resonator behavior. We apply this theory specifically to an electrically injected, coherently coupled, 1 × 2 vertical cavity surface emitting laser (VCSEL) array.

While the study of PT-symmetric Hamiltonians traces to the seminal paper in 1998 [24], coherently coupled semiconductor laser arrays have been studied for more than three decades [25–33] in various applications, including high-brightness beam generation [34–37], beam steering [26,32,38,39], and high bandwidth modulation [40,41]. The coupled-mode theory has been used to describe the supermodes of the array [28,42,43], phase velocity matching [44], and dynamics [32,45]. However, in previous coupled-mode analyses of coherent laser arrays, the possibility of gain differences between the individual resonators was often not included or was implicit in a set of carrier density equations coupled to the optical field equations [32,46]. The analysis here takes gain in the individual resonators as an explicit parameter (imaginary part of the complex frequency) instead of introducing coupled carrier density equations. This preserves the linearity of the coupled-mode theory while still offering insight into salient physical effects.

The effects of the gain/loss contrast, frequency detuning, and their interplay on the supermodes of coherent 1 × 2 VCSEL arrays are discussed. It is shown below that beam steering in coherently coupled lasers is a direct result of the optical non-Hermiticity of the system caused by the gain/loss contrast and that the π/2 limit of the maximum relative phase tuning [26,47,48] is reached at or beyond the exceptional point in the PT-symmetry broken regime. This beam steering mechanism of directly coupled resonators is essentially different from the case where the resonators are locked to a common master [49,50] or in optical nano-antenna arrays [51]. Experimentally, we demonstrate the evolution of the lasing supermode, including beam steering, near-field intensity distribution, hopping of the lasing mode between in-phase and out-of-phase modes, and PT symmetry breaking. We also show our experimental measurements, including the operating wavelength, relative intensities, and relative phase, agree quantitatively with the temporal coupled-mode theory.

2. TEMPORAL COUPLED-MODE ANALYSIS OF COHERENTLY COUPLED 1 × 2 VCSEL ARRAYS

In previous experimental studies on electrically injected coherently coupled laser arrays, current-controlled frequency tuning
or phase velocity tuning was shown to be critical for mode control [26,32,33,38,44]. However, the effect of the gain contrast was rarely mentioned in those studies. On the other hand, the effects of the gain/loss contrast in optically pumped coupled waveguides [6,8,52,53] and coupled ring lasers [18,54], coupled quantum cascade lasers (QCLs) [23], and in general photonic molecules [55] have been studied extensively recently, but the frequency or phase velocity detuning was often negligible or constant. Here, we apply the temporal coupled-mode theory to electrically injected coherently coupled VCSEL arrays, where frequency tuning and gain tuning are not independent. This section discusses how the coupled mode evolves with the tuning of both the gain and frequency and compares it with the cases of only frequency detuning or gain contrast, with a focus on the experimental observables in 1×2 VCSEL diode arrays: wavelength, modal gain, intensity, and phase distribution of the coupled modes.

Given two resonant modes that are weakly coupled, the amplitudes \( E_a \) and \( E_b \) of the two modes can be described by

\[
d\frac{dE_a}{dt} = -i\omega_a E_a + \gamma_a E_a + i\kappa_{ab} E_b,
\]

\[
d\frac{dE_b}{dt} = -i\omega_b E_b + \gamma_b E_b + i\kappa_{ba} E_a,
\]

where \( \omega_{a,b} \) are the natural resonant frequencies of the two modes, \( \kappa_{ab,ba} \) are the coupling coefficients, and \( \gamma_{a,b} \) are the temporal gain/loss coefficients (a positive value represents gain, and a negative value represents loss). Weak coupling means that \( \kappa_{ab,ba} \ll \omega_{a,b} \).

Here, we assume that all the coefficients, \( \omega_{a,b}, \gamma_{a,b}, \) and \( \kappa_{ab,ba} \), are constants, which forces the system to be linear and ignores all the non-linear effects, such as gain saturation or dynamic coupling between field amplitudes and carrier densities. This approximation can be justified by either considering only small field amplitudes at or below the threshold, where the gain is not saturated, or by assigning the gain coefficient to be the saturated value, given the knowledge of the field amplitudes. Equation (1) can be written in compact form as

\[
d\frac{dE}{dt} = \Omega E,  \tag{2}
\]

where

\[
\begin{pmatrix}
E_a \\
E_b
\end{pmatrix}
\quad \Omega =
\begin{pmatrix}
\omega_a + i\gamma_a & \kappa_{ab} \\
\kappa_{ba} & \omega_b + i\gamma_b
\end{pmatrix}.
\]

Equation (2) has the same form as the Schrödinger equation in quantum mechanics, where \( \Omega \) is an optical analogy to the Hamiltonian of a quantum particle inhabiting a coordinate axis that consists of just two points [56]. The action of the operator PT as an analogy to the quantum particle can be defined as

\[
(PT)_{ij} = \Omega^*_{i,j} \Omega_{j,i} \tag{3}
\]

where \( i, j \) are the matrix indices taking the values of 1,2, and hence, a general PT-symmetric \( \Omega \) can be expressed as

\[
\begin{pmatrix}
\begin{array}{cc}
\chi & \gamma \\
\gamma^* & \chi^*
\end{array}
\end{pmatrix}
\]

In a power-conserving, lossless, gainless, coupled-resonator system, \( \kappa_{ab} = \kappa_{ba}, \gamma_a = \gamma_b = 0 \), which means \( \Omega \) is Hermitian (and PT symmetric if \( \omega_a = \omega_b \)). If balanced gain and loss are judiciously introduced (i.e., \( \gamma_a = -\gamma_b \)), the system is no longer Hermitian, but it can still be PT symmetric if \( \omega_a = \omega_b \). Note that the condition of balanced gain and loss, \( \gamma_a = -\gamma_b \), can be relaxed to any gain or loss contrast, as long as \( \gamma_a \neq \gamma_b \), if we change variables to extract a common gain/loss factor \( \exp(\gamma_a + \gamma_b)t/2 \) from the temporal dependence of the field amplitudes [5,54].

Demanding that the field amplitudes are time harmonic \( E = (A_a/A_b)e^{i\omega t} = A_be^{-i\omega t} \), Eq. (2) becomes an eigenvalue problem, \( \Omega A = \omega A \). The eigenvalue \( \omega \) represents the complex frequency of the coupled modes, with the real part representing the angular frequency and the imaginary part representing the gain/loss coefficient. The eigenvector \( A = (A_a/A_b) \) represents the composition of the supermodes, which is a superposition of the two individual modes. The solution of the eigenvalue equation is

\[
\omega = \frac{\omega_a + \omega_b}{2} + \frac{i(\gamma_a + \gamma_b)}{2} \pm \sqrt{\left(\kappa_{ab}\kappa_{ba} + \left(\frac{\omega_a - \omega_b}{2}\right)^2 - \left(\frac{\gamma_a - \gamma_b}{2}\right)^2 \right)^2 + \left(i(\omega_a - \omega_b)(\gamma_a - \gamma_b)/2\right)^2}^{1/2}. \tag{4}
\]

\( A \) can be calculated from the eigenfrequencies by

\[
\frac{A_b}{A_a} = \frac{-i\kappa_{ba}}{i(\omega_a - \omega_b) + \gamma_b}. \tag{5}
\]

The magnitude of the ratio, \( |A_b/A_a| \), controls the near-field intensity profile of the mode, while the phase of the ratio, \( \text{Arg}(A_b/A_a) \), determines the relative phase tuning, which leads to beam steering in the far field [48]. Equations (4) and (5) take simpler forms when there is only gain contrast or frequency detuning, as discussed in detail in [6,8,15,44,52,55,57], for example. Here, we apply some of those ideas in the context of coupled VCSEL diode arrays as a comparison to our subsequent discussion of simultaneous gain and frequency tuning. For simplicity, we set the general complex coupling coefficient to be symmetric, real and positive, i.e., \( \kappa_{ab} = \kappa_{ba} = \kappa > 0 \), assuming negligible deviation in the coupling coefficients from the case of two identical passive resonators.

When the two resonators of the array have different native resonant frequencies but experience no gain/loss contrast, i.e., \( \omega_a - \omega_b \neq 0, \gamma_a = \gamma_b \), the wavelengths, modal gain, and field amplitude ratio (both magnitude and phase) of the coupled modes are illustrated in Fig. 1. The frequency detuning changes the coupled-mode intensity distribution, such that the out-of-phase mode has more intensity in the resonator with the higher natural resonant frequency, while the in-phase mode has more intensity in the cavity with the lower natural resonant frequency. The degree of intensity distribution asymmetry increases with the frequency detuning, as shown in Fig. 1(c). The fact that the eigenvector \( A \) is purely real for a Hermitian \( \Omega \) means no phase tuning or beam steering is induced if gain contrast is absent, as shown in Fig. 1(d).

When the two resonators of the array have identical native resonant frequencies but experience gain contrast, i.e., \( \omega_a = \omega_b \), \( \gamma_a - \gamma_b \neq 0 \), it is found that \( \Omega \) is not Hermitian, but PT symmetric. In this case, the evolution of the coupled modes with varying gain contrast is illustrated in Fig. 2.

The fact that the phase tuning cannot exceed the limit of \( \pm \pi/2 \) has been reported in coupled laser array experiments; see, for example, [26,47,48]. Upon hitting the \( \pi/2 \) phase tuning limit, further increasing the gain contrast results in driving the array into the PT-symmetry broken regime, where the relative...
phase is pinned at $\pi/2$ and the intensity distribution of the coupled modes becomes asymmetric. It has been observed in previous experiments that the mutual coherence between the cavities decreases when the $\pi/2$ phase tuning limit is reached [47,48]. This loss of mutual coherence can be expected, as the coupled modes become asymmetric and spatially concentrate in single cavities, resulting in the simultaneous lasing of both coupled modes. Another feature worth noting in Fig. 2 is the appearance of exceptional points or branching points of $\Delta g/\kappa = 0.136/0.0006^2$, where the two eigenmodes collapse. Because of this collapse of eigenmodes, in Figs. 2(b) and 2(c) the modes are not labeled as in-phase or out-of-phase to avoid confusion, because unlike the case in the Fig. 3, the coupled modes cannot be traced back across the exceptional point to be identified as in-phase or out-of-phase modes.

When both gain contrast and frequency detuning exist, the coupled modes are controlled by the interplay of the frequency detuning and gain contrast. For example, it has been shown that the undesired deviation from the exceptional point caused by a complex coupling coefficient can be “healed” by adding a real index variation to the gain/loss contrast [14,53]. In coupled VCSEL diode arrays, both the frequency detuning and the gain contrast are driven by asymmetric current injections into the resonators of the array, and they are both linearly dependent on the injection current difference [58,59]. The evolution of the eigenmodes in this situation is illustrated in Fig. 3. The degeneracies we see in the ideal PT-symmetric case (Fig. 2) do not exist when frequency detuning is present. Also note that the gain of the (skewed) in-phase mode is higher than the (skewed) out-of-phase mode when the current injection difference is non-zero. This is because the change of the intensity distribution of the in-phase mode, as a result of simultaneous frequency detuning and gain contrast, enhances its spatial overlap with the spatially non-uniform gain, while the intensity distribution changes of the out-of-phase mode do the opposite. Whether it is the in-phase mode or out-of-phase mode that gets the higher gain depends on the sign of $\omega_a - \omega_b/\gamma_a - \gamma_b$. For the out-of-phase mode to have the higher gain, the local resonant frequency must increase with the increasing local gain: for example, if the carrier-induced index suppression dominates the thermal effect. It has been known that evanescently coupled VCSEL arrays tend to work in the out-of-phase mode due to less spatial overlap with the lossy inter-element area, although for most applications, the in-phase mode is preferred. The gain discrimination preference for the in-phase mode suggests that with a sufficiently large current injection difference, the mode may hop from the out-of-phase mode to the in-phase mode, offering a novel modal

![Fig. 1. Effect of frequency detuning without gain contrast on (a) wavelengths of the coupled modes, (b) gain of the coupled modes, (c) ratio of the field magnitudes in two cavities, and (d) relative phase between the fields in two cavities.](image1)

![Fig. 2. Effect of gain contrast without frequency detuning on (a) wavelengths of the coupled modes, (b) gain of the coupled modes, (c) ratio of the field magnitudes in two cavities, and (d) relative phase between the fields in two cavities.](image2)

![Fig. 3. Effect of coexisting gain contrast and frequency detuning on (a) wavelengths of the coupled modes, (b) gain of the coupled modes, (c) ratio of the field magnitudes in two cavities, and (d) relative phase between the field in two cavities. It is assumed that the local changes of the gain and frequency are both linearly dependent on the current difference, with $\Delta g = -4\Delta \omega$, and the maximum gain contrast at the edge of the graphs is $\Delta g_{\text{max}} = 4\kappa$.](image3)
control method and reconfigurability. This mode hopping behavior is observed experimentally and discussed in the experimental section below. Figure 3(d) also shows that the phase tuning limit is less than $\frac{\pi}{2}$. Hence, to achieve the theoretical limit of $\frac{\pi}{2}$ phase tuning, one must minimize the frequency detuning accompanying non-uniform pumping.

3. CHARACTERIZATION OF COHERENTLY COUPLED 1 x 2 VCSEL ARRAYS

The above analysis can be applied to 1 x 2 coherently coupled photonic crystal VCSEL arrays, which are shown in Fig. 4(a) [33,37]. The electrical apertures of the VCSELs are defined by proton ion implantation, and a focused ion beam (FIB) etch between the two cavities improves the electrical isolation, enabling individual control of the injected currents [60]. Lateral optical confinement is provided by photonic crystal patterns etched into top facet, leading to single-mode operation for each element VCSEL. Details of the device structure and fabrication process have been described in [33]. In coupled VCSEL arrays, the eigenvector $A$ of the lasing coupled mode can be characterized from near-field and far-field measurements, as shown in Figs. 4(b) and 4(c) [48]. The relative phase between the two cavities can be extracted by propagating the near field with the assumed phase, checking the propagated far field with the measured far field, and iterating with a better assumed phase if the far fields do not agree [48]. Figure 4(d) shows the agreement of the propagated and measured far fields. Figure 5 is a summary of the measured coupled-mode wavelength, near-field amplitude ratio, and extracted relative phase compared with the coupled-mode theory. As shown in the bottom of Figs. 4(b)–4(d), the lasing mode starts as a slightly skewed out-of-phase mode with equal injected currents $I_a = I_b = 3.99$ mA. Then, as $I_b$ is decreased to $I_b = 3.87$ mA, we see the minimum of the far field moves away from the on-axis center as a result of beam steering. At $I_b = 3.86$ mA ($\Delta I = -0.13$ mA), the lasing mode hops to the (skewed) in-phase mode, indicated by the abrupt change in the relative phase [Fig. 5(d)], wavelength [Fig. 5(a)], and the near-field intensity distribution [Fig. 5(c)], which are marked by the red arrows in Figs. 4 and 5. $I_b$ was then decreased to 3.83 mA and subsequently increased to 3.99 mA, with the reverse mode hopping happening at $\Delta I = -0.08$ mA, marked by the blue arrows in Figs. 4 and 5. Hysteresis and bistability are observed in the region of $-0.13 \text{ mA} < \Delta I < -0.08 \text{ mA}$. We cannot measure the gain of the coupled modes directly, but the mode hopping is an indication of the change in the modal gains of the respective modes. The measured intensity ratio data appear to be offset from the theory in Fig. 5(c). This is because the two array elements were not identical, where the output of laser $a$ was stronger than laser $b$ even with the same injected current, which is believed to be a result of fabrication imperfections (for example, the slightly off-center FIB etch apparent in Fig. 4(a)).

As discussed previously, PT-symmetry breaking can be identified by the pinning of the phase to $\frac{\pi}{2}$. For the device shown in Figs. 4 and 5, the maximum relative phase appears to be limited to $0.4\pi$ as a result of the frequency detuning expected to accompany selective electrical pumping. If the constituent VCSEL elements of the array initially exhibit a native frequency splitting that was eliminated by the effects of selective electrical pumping, we would expect to see a closer approach to the theoretical maximum phase difference. For a different VCSEL array that we have

Fig. 4. (a) Microscopic photo of the VCSEL diode array under characterization, (b) near-field intensity profiles with extracted relative phase [48], (c) far-field intensity profiles measured at $\Delta I = 0$, $-0.12$, and $-0.09$ mA, and (d) evolution of the measured and propagated far-field profiles with varying $I_b$ and fixed $I_a$. The red circles in (c) denote a 12-deg angular spread in the far field.
characterized, the $\pi/2$ relative phase was obtained, with near-field and far-field intensity profiles shown in Figs. 6(a) and 6(b). These measurements are in good agreement with the simulated broken PT-symmetry eigenstate shown in Figs. 6(c) and 6(d). For this particular coupled array, as a result of fabrication imperfections, the two VCSEL elements are spectrally aligned when the gain contrast was sufficiently large for PT-symmetry breaking.

4. CONCLUSIONS

In summary, electrically injected optically coupled VCSEL arrays are well modeled using the time-domain coupled-mode theory, allowing us to account for the gain contrast and frequency splitting and predict a range of physical behaviors associated with PT symmetry and symmetry breaking. We have observed PT-symmetry-related mode evolution and PT-symmetry breaking in room temperature coupled VCSEL diode arrays with quantitative agreement with the theory. A controllable mode hopping between the in-phase and out-of-phase modes is identified here for the first time. This work demonstrates the potential of designing the gain/loss profile for non-Hermitian engineering of coupled optical systems and may lead to new engineering applications in the future.

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