Non-Hermiticity and exceptional points in coherently coupled vertical cavity laser diode arrays

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ABSTRACT

Coherently coupled laser arrays can be described by the temporal coupled mode theory in which the array modal behavior can be classified according to the coupling matrix, $\mathbf{M}$. Accounting for a nonuniform gain/loss distribution in a laser array makes $\mathbf{M}$ a non-Hermitian matrix, and experimentally we find phase-front tuning (beam steering) of the coherent supermode as a result of the non-Hermiticity. We report the experimental characterization of the supermodes in coherently coupled vertical cavity surface emitting laser diode arrays and demonstrate control of non-Hermiticity by spatially varying injection currents. Exceptional points are identified in these electrically injected microcavity diode arrays.

Two coupled resonators can be described by the temporal coupled mode theory:

$$\frac{d\mathbf{E}}{dt} = -\mathbf{M}\mathbf{E},$$

where $\mathbf{E} = (E_A, E_B)$ and $\mathbf{M} = \begin{pmatrix} \omega_A + i\gamma_A & -\kappa \\ -\kappa & \omega_B + i\gamma_B \end{pmatrix}$, in which $E_{A,B}$ are the slowly varying complex field amplitudes in the two cavities, $\omega_{A,B}$ are the local resonant frequencies, and $\gamma_{A,B}$ are the local...

Coherently coupled semiconductor laser arrays have been experimentally studied for almost 5 decades. Electronic control of the lasing mode, including electronic beam steering, is among the earliest observations from diode laser arrays. Beam steering (off-normal-angle emission) in the far field originates from the phase profile (tilted phase front) in the near field, which is controlled by the nonuniform gain/loss profile in the array. Recently, the interest in the control of the gain/loss profile has been revived in the context of parity time (PT) symmetry and non-Hermitian photonics. Optically coupled and electrically isolated vertical cavity surface emitting laser (VCSEL) diode arrays possess unique advantages for the study of gain/loss engineering and non-Hermitian photonics. Besides being a technologically mature platform, the gain/loss profile in such arrays can be conveniently controlled using the frequency detuning between the two cavities, as revealed recently.

In this paper, we first introduce non-Hermiticity in the context of a coupled laser system using temporal coupled mode theory. Then, we describe our coherently coupled $2 \times 1$ VCSEL arrays and present an overview of mode characterizations that we have done, while the two injection currents ($I_A$ and $I_B$) are independently controlled. We present electronical steering of the far field emission angle as a consequence of the gain/loss tuning (non-Hermiticity) in the array. Light output power, near-field profiles, far-field profiles, and the emission spectra are measured as a function of both injection currents, $I_A$ and $I_B$. We also provide a brief review of the mode tuning mechanism to help understand the control of non-Hermiticity via differential current injection. At last, we look into the non-Hermiticity and identify two observations that we highlight. Firstly, we identify exceptional points in these electrically injected diode laser arrays and report abrupt mode switching behavior around the exceptional point. Secondly, we show that the imaginary component of the coupling coefficient is a source of non-Hermiticity, and it can be identified from the power increase due to coherent coupling.

Two coupled resonators can be described by the temporal coupled mode theory:
gain/loss coefficients, and \( \kappa \) is the coupling coefficient. We have assumed \( \mathbf{M} \) to be a symmetric matrix (\( \mathbf{M}^T = \mathbf{M} \)) by assuming the coupling coefficients to be symmetrical, which is a first order approximation for two resonators that are identical except for frequency detuning.\(^{24} \)

The coupled modes, or supermodes, of the coherent array can be calculated by solving for the eigenvalues and eigenvectors of \( \mathbf{M} \). The eigenvalues of \( \mathbf{M} \) are the frequencies of the coupled modes, whereas the eigenvectors represent the field distributions. Coupled resonator arrays can be classified according to \( \mathbf{M} \). Matrix \( \mathbf{M} \) (and hence, the system) can have either Hermitian or non-Hermitian properties. For example, in a passive and lossless system, \( \mathbf{M} \) is Hermitian (\( \mathbf{M}^H = \mathbf{M} \)).\(^{27} \) A symmetric and Hermitian matrix is a real-valued matrix with real-valued eigenvectors, meaning that the relative phase between fields in two resonators is fixed at 0 or \( \pi \) (i.e., no relative phase tuning).

Supermode tuning that involves relative phase variation between the elements originates from the non-Hermiticity of \( \mathbf{M} \), and thus is a concise manner for describing the gain/loss engineering that translates into the phase front engineering of the array output beam. The non-Hermiticity of \( \mathbf{M} \) is also connected to other intriguing properties of the array, for example, improved side-mode suppression,\(^{22,23} \) asymmetrical energy flow between resonators,\(^{30} \) as well as enhanced functionality around the exceptional points.\(^{7,31} \) For the latter, exceptional point operation from an electrically injected laser diode array is desired.\(^{31} \) Previously, exceptional points have been identified in coupled laser systems including optically pumped coupled ring lasers,\(^{32} \) whispering gallery microlasers,\(^{33,34} \) and quantum cascade lasers.\(^{35,36} \) Here, we show the experimental identification of exceptional points in an electrically injected coupled laser system.

A cross-sectional sketch and a top image of a 2 \( \times \) 1 coherently coupled VCSEL array are shown in Fig. 1. The optical cavities of the two lasers are defined by the two epitaxial distributed Bragg reflector (DBR) mirrors in the longitudinal direction and by the photonic crystal (PhC) patterns in the transverse directions.\(^{37} \) The electrical apertures confining the injected currents into the array elements are defined by multiple steps of ion-implantations with different energies.\(^{28} \) We find that currents \( I_A \) and \( I_B \) can be independently varied with \( >1 \text{M}\Omega \) electrical isolation between the electrodes contacting the cavities.

Because \( I_A \) and \( I_B \) can be independently tuned, the characterization of two element coherent VCSEL arrays can be conveniently represented with 2-dimensional graphs with \( I_A \) and \( I_B \) as the coordinate axes, and the measured quantity being represented by a color scale. For example, Fig. 2(a) shows the output power versus injection current \( I_A \) and \( I_B \), and the color scale represents the output optical power at each point \((I_A, I_B)\). Figures 2(b) and 2(c) similarly show the far-field interference visibility and the beam-steering angle that results from the mutual coherence (phase-locking) and phase tuning between the two cavities.

When the two VCSEL elements in the array are phase-locked, we observe an interference pattern in the far field. The visibility of the interference pattern, defined as \( \text{visibility} = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \), is approximately the magnitude of the mutual coherence between the two lasers when their individual intensities are not too different.\(^{29,30} \) Unitary visibility corresponds to perfect mutual coherence, while zero visibility implies completely incoherent and spatially distinct modes emitted from the elements. Visibility between zero and one is partial coherence, which can arise as a result of the coexistence of in-phase and out-of-phase coupled modes.\(^{23} \) We plot the far-field interference visibility versus the two injection currents in Fig. 2(b), which clearly elucidates the locking region along the diagonal of the plot where the interference visibility becomes close to 1. Within the locking region, the relative phase between the two lasers can be tuned by injection currents, leading to beam steering.\(^{3,32} \)

Shown in Fig. 2(c) is the plot of beam steering angle (i.e., the angle of the far-field intensity maximum) versus injection currents. The phase tuning mechanism is unique to weakly coupled diode laser arrays, as recently explained in Ref. 10. This tuning mechanism is important in interpreting the experimental results we report here, so we briefly described it in the following.

The injection currents vary the carrier injection rates into the cavities and also tune the resonance frequencies of the cavities through Joule heating and the temperature dependence of the refractive index in semiconductors.\(^{21} \) The optical spectra of the array are measured using an optical spectrum analyzer (OSA) with 0.02 nm spectral resolution. Shown in Fig. 2(d) is a plot of the lasing wavelength when \( I_A \) is fixed and \( I_B \) is varied through the locking region. Within the locking region, the spectrum shows a single peak, corresponding to the coherent coupled mode, while out of the locking region, the two lasers in the array operate in independent localized modes with distinct wavelengths.\(^{22,30} \) While increasing \( I_B \) mostly increases the temperature (and hence the wavelength) of laser B in Fig. 2(d), there is a small degree of thermal crosstalk evident in the wavelength shift of laser A. The cavity frequency detuning \( (\Delta \Omega = \Omega_B - \Omega_A) \) varies linearly with \( \Delta I = I_B - I_A \), as evident in Fig. 2(d). Note that \( \Omega_{A,B} \) are defined as the cavity resonance frequencies when the carrier densities of the individual cavities are at the threshold, and they are identical to \( \Omega_{A,B} \) when the lasers are not coupled, because of carrier density clamping. However, when the lasers are coupled, the cavity frequency detuning \( \Delta \Omega \) can be different from the total frequency detuning \( (\Delta \Omega = \omega_{\text{th}} - \omega_{\text{in}}} \), because \( \Delta \) includes the contribution of detuning from the potentially asymmetrical carrier density distribution. We define \( \Delta \) in this way to represent the part of frequency detuning that is experimentally linearly controlled, while \( \Delta \) is not directly

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**FIG. 1.** (a) Sketch and (b) scanning electron microscopy (SEM) image of a 2 \( \times \) 1 coherent VCSEL array.
experimentally controlled. When the two VCSELs are coherently coupled (phase-locked), the relative phase between two cavities is controlled by $\Delta \Omega$. This is shown in Fig. 2(c). The cavity frequency detuning $\Delta \Omega$ controls the gain/loss ($A_B$; $B$) in the individual lasers through a unique nonlinear mechanism that involves gain saturation and the linewidth enhancement factor. To summarize, we experimentally control $\Delta \Omega$ through local thermal tuning, and $\Delta \Omega$ is proportional to the difference in current injection ($I_B - I_A$). Through the nonlinearities in semiconductor lasers, the cavity detuning $\Delta \Omega$ induces asymmetrical carrier density distribution and hence the gain/loss in the cavities, which results in the tuning of non-Hermiticity and the relative phase between cavities. Because of the carrier induced frequency shift in semiconductor lasers, $\Delta \Omega$ can often be different from $\Omega$ as discussed in Ref. 10.

When the two lasers are phase-locked, they operate in a single coherent supermode. The coherent supermode extends into both optical cavities, and is more efficiently pumped as compared to the two independent and incoherent modes. Evidence of this is apparent along the diagonal of Fig. 2(a) above the lasing threshold. The “ridge” of increased output power along the diagonal of Fig. 2(a) arises from the decrease in the threshold that occurs when the two currents are approximately equal and the lasers phase-lock. This increase in output power (decrease in the lasing threshold) due to non-conservative coupling is a quantitative observation of the gain splitting represented by the imaginary coupling component, $\kappa$, which is another source of non-Hermiticity.

Different mode tuning behavior is observed when the array is biased near the threshold, as compared to a higher bias. Figures 3 and 4 show far-fields, near-fields, and the extracted coherent modes when the coherent array is biased near the thresholds ($\lambda I_B$), while Figs. 5 and 6 are at a higher bias level ($\lambda I_B$). The coherent mode can be expressed as $E = \frac{1}{\sqrt{R}} e^{i \phi}$, where $R$ is the near-field intensity ratio and $\phi$ is the relative phase. From the near-field intensity measurements, we can extract $R$. From the relationship between the far field and the near field, we can also experimentally extract $\phi$ and the magnitude of mutual coherence. The summary of $R$, $\phi$, and the degree of coherence are shown in Figs. 4 and 6. Near the threshold, both $R$ and $\phi$ are found to be tuned by the current. At high bias levels, $R$ remains close to 1 and invariant against current tuning, consistent with coupled modes that resemble $E = \frac{1}{e^{i \phi}}$ as predicted by PT symmetric non-Hermiticity.

Comparing Figs. 4 and 6, we also see that the phase tuning is more sensitive to the current tuning at higher bias levels. Because the amount of frequency detuning induced by current tuning is measured to be almost the same between low bias ($\lambda / \Delta I = 0.460 \text{ nm/mA}$ near 3.8 mA) and high bias ($\lambda / \Delta I = 0.468 \text{ nm/mA}$ near 5 mA), the more sensitive phase
tuning at a higher bias suggests that the coupling coefficient (more precisely, the factor $\lambda_i + \lambda_j$) is smaller at a higher bias, which is consistent with the smaller locking region.

FIG. 3. Tuning of the coherent mode near threshold ($\sim 1.1 I_{th}$): (a) far field and (b) near field measurements. The relative phase between two lasers is extracted iteratively by comparing the measured far field [solid blue line in (a)] and the one propagated from the near field [dashed red lines in (a)]. $I_B$ was fixed at 3.5 mA.

FIG. 4. Summary of the extracted relative phase, the degree of coherence, and the near field intensity ratio near threshold ($\sim 1.1 I_{th}$). $I_B$ was fixed at 3.5 mA.

FIG. 5. Tuning of the coherent mode above the threshold ($\sim 1.4 I_{th}$): (a) far field and (b) near field measurements. The exceptional point is labeled as EP. The relative phase between two lasers is extracted iteratively by comparing the measured far field [solid blue line in (a)] and the one propagated from the near field [dashed red lines in (a)]. $I_B$ was fixed at 4.6 mA.

FIG. 6. Summary of the extracted relative phase, the degree of coherence, and the near field intensity ratio above the threshold ($\sim 1.4 I_{th}$). The exceptional point is labeled as EP. $I_B$ was fixed at 4.6 mA.
From the perspective of searching for exceptional points, we know from the coupled rate equation analysis that the exceptional points are located on the boundaries of the locking region under the zero-detuning condition ($\Delta \omega = 0$).\(^{20,29}\) However, as discussed previously, we cannot directly experimentally measure when $\Delta \omega = 0$ in the system and because of nonlinear coupling, this condition can experimentally occur for $\Delta I \neq 0$ and $\Delta \Omega \neq 0$. Exceptional points would be identified from modes with balanced intensity distribution and $\pi/2$ relative phase (i.e., $R = 1$, $\phi = \pm \pi/2$).\(^{20,29}\) We also would need to show that the mode is a pure supermode, and thus would need the far-field visibility to be unitary. The operation point (vii) in Figs. 5 and 6 shows an example of the exceptional point identified in this manner. The optical mode is extracted to be $(1, -i)$, and the interference in the far field shows almost perfect mutual coherence between cavities (i.e., a pure supermode). We observe abrupt mode switching around the exceptional point, as evident in the abrupt switching of the relative phase, and the sharp decrease in the degree of coherence as current detuning increases beyond the exceptional point. It would be preferential to have independent control of the cavity detuning and the injection rate, but in our current devices, we are controlling both of them with the same control dial $\Delta I$. For example, in Fig. 4, we do not identify exceptional points because we cannot control the frequency detuning and the gain contrast independently. However, it is unambiguous that the operation point (vii) in Fig. 5 is an exceptional point (or very close to an exceptional point), because of the balanced laser intensity between two cavities and the $\pi/2$ relative phase. The automation in characterization and data processing has facilitated this search.

In conclusion, we show that coherently coupled $2 \times 1$ VCSEL arrays can be described by a non-Hermitian coupling matrix, $\overline{M}$. Relative phase tuning and the existence of exceptional points can be viewed as manifestations of the non-Hermiticity of $\overline{M}$. We present thorough (both $I_x$ and $I_y$ are varied) characterization of the array output power, the far-field visibility, the beam steering angle, and the emission spectra with frequency detuning. Close to the threshold, the injection current tunes both the relative intensity and the relative phase between cavities. At higher bias levels, the array exhibits only phase tuning without intensity tuning, and exceptional points with abrupt mode switching can be identified. Finally, we show that the magnitude of the complex coupling coefficient between the elements decreases at higher bias levels, leading to a smaller locking range and an increase in phase tuning sensitivity. With this new perspective on the operation of the coherent VCSEL arrays, we are pursuing enhanced functionality and applications.

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